Projects SPA 2019

Khalil Ghorbal

If you still want to propose your preferred problem, please come forward. You still have till **Monday October 14th**.

Otherwise, by the same date, you have to pick up a problem from the list below. The list is extracted from the famous list by Garey and Johnson [1] which you can find almost entirely here `https://en.wikipedia.org/wiki/List_of_NP-complete_problems`.

Each problem is separated into an instance describing the given data and the question or problem of interest. Problems marked with (∗) are not known to be NP-complete (and are therefore NP-hard). The last three problems are open in the sense they are not known to be NP-complete or in P (or neither).

Once a problem is selected, your task will be to

- Understand and document the problem (known and recent related results)
- Apply at least two out of three solvers we've seen during the course to solve generic instances. Your implementation should reasonably work at least for a class of small instances of the problem
- Report your findings and results in a paper that you will be submitting online

Clique Instance: a graph $G = (V, E)$, a positive integer $K \leq |V|$. Problem: does G contain a clique of size K or more. i.e., a subset $V' \subseteq V$ with $|V'| \geq K$ such that every two vertices in V' are joined by an edge in E?

Directed Elimination Ordering Instance: directed graph $G = (V, A)$, non-negative integer K. Problem: is there an elimination ordering for G with fill-in K or less, i.e., a one-to-one function $f : V \to \{1, 2, \ldots, |V|\}$ such that there are at most K pairs of vertices $(u, v) \in V \times V \setminus A$ with the property that G contains a directed path from u to v that only passes through vertices w satisfying $f(w) < \min\{f(u), f(v)\}$?

Consecutive Ones Submatrix Instance: An $m \times n$ matrix A of 0's and 1's and a positive integer K. Problem: Is there an $m \times K$ submatrix B of A that has the *consecutive ones* property, i.e., such that the columns of B can be permuted so that in each row all the 1's occur consecutively?

Quadratic Programming (*) Instance: finite set X of pairs $(\bar{x}, b)$, where $\bar{x}$ is an m-tuple of rational numbers and b is a rational number, two m-tuples $\bar{c}$ and $\bar{d}$ of rational numbers, and a rational number B. Problem: Is there an m-tuple $\bar{y}$ of rational numbers such that $\bar{x} \cdot \bar{y} \leq b$ for all $(\bar{x}, b)$ in X and such that $\sum_{i=1}^{m}(c_i y_i^2 + d_i y_i) \geq B$, where c_i, y_i, and d_i denote the ith components of $\bar{c}$, $\bar{y}$, and $\bar{d}$ respectively?

Algebraic Equations Over $\mathbb{F}_2$ Instance: Polynomials P_i, $1 \leq i \leq m$, over $\mathbb{F}_2[X_1, \ldots, X_n]$. Problem: do all polynomials P_i have a common solution in $\mathbb{F}_2$?

Quantified Boolean Formulas (QBF) (*) Instance: set $X = \{x_1, \ldots, x_n\}$ of variables, well-formed quantified Boolean formula $F = (Q_1 x_1) \cdots (Q_n x_n) E$, where E is a Boolean expression and each $Q_i \in \{\exists, \forall\}$. Problem: is F true?

Graph Isomorphism Instance: two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. Problem: are G_1 and G_2 isomorphic, i.e., is there a one-to-one onto function $f : V_1 \to V_2$ such that $\{u, v\} \in E_1$ if and only if $\{f(u), f(v)\} \in E_2$.

Graph Genus Instance: graph $G = (V, E)$ and a non-negative integer K. Problem: Can G be embedded on a surface of genus K such that no two edges cross one another?

Linear Programming Instance: Integer-valued vectors $\bar{v}_i$, $1 \leq i \leq m$, $\bar{d} = (d_1, \ldots, d_n)$, and $\bar{c}$, and an integer b. Problem: is there a vector $\vec{x}$ of rational numbers such that, for $1 \leq i \leq m$, $\bar{v}_i \cdot \bar{x} \leq d_i$, and $\bar{c} \cdot \vec{x} \geq b$?

References

[1] D. S. Johnson M. R. Garey. *Computers and Intractability: A Guide to the Theory of Np-Completeness.* W H Freeman Worth Pub 3PL, 1979.