Extracting Information from Moments: Christoffel Polynomial and Gibbs Phenomenon

Research Context. Certification and validation of computational results is a major issue for modern sciences raising challenging problems at the interplay of mathematics and computational aspects of computer science. One can emphasize in this context several applications arising in the design of modern cyber-physical systems with a crucial need of certification and information extraction. In particular, one tries to avoid incidents such as the Patriot missile crash in 1991, the FDIV Pentium bug in 1994 or more recently the collision of Google's self-driving car in 2016.

These issues give rise to many mathematical problems. Polynomial optimization (which consists in computing the infimum of a polynomial function under algebraic constraints) is one of the most important, difficult and challenging one. The emergence of this exciting new field goes back to the last decade and has led to striking developments from a cross fertilization between (real) algebraic geometry, applied mathematics, theoretical computer science and engineering. The backbone of this powerful methodology is the "moment-SOS" approach, also known as "Lasserre hierarchy".

This research project aims at designing polynomial optimization algorithms to extract useful information from *moment* data.

Goals In the context of global optimization, we are concerned with the design of *solution extraction* procedures, providing for instance, the minimizer(s) of a given polynomial. For a finite set of minimizers, this corresponds to the support of a discrete measure. To improve upon existing extraction procedures, we intend to rely on the *Christoffel polynomial* [1] associated to the matrix whose entries are the moments of such a measure. Previous research focused on approximating the set of minimizers for linear functionals coming from positive measures (e.g. convex combination of Dirac measures). To find the set of minimizers of general nonnegative polynomials, we intend to study the Christoffel function associated to linear functionals coming from *signed* measures.

Several applications lead to approximate functions corresponding to measures supported on indicator functions or trajectories. However it is typically hard to approximate them with polynomials of increasing degrees in order to obtain a pointwise convergence behavior. Such non-uniform convergence behaviors are related to an effect known as the *Gibbs phenomenon*. Another goal of this project is to explain and attenuate this phenomenon. The starting point will be to investigate classical optimization problems, with available solutions.

Working Context The internship will be co-advised by Jean-Bernard Lasserre (CNRS LAAS) and Victor Magron (CNRS L2S CentraleSupelec / CNRS LAAS). The Master student will be hosted by the Mac team in the LAAS laboratory, located at Toulouse.

Required Skills Motivated candidates should hold a Bachelor degree and have a solid background in **either** optimization, signal processing, control, real algebraic geometry or computer algebra. Good programming skills are also required. The candidates are kindly asked to send an e-mail with "M2 candidate" in the title, a CV and motivation letter to `lasserre@laas.fr` and `victor.magron@l2s.centralesupelec.fr`. Knowledge of French does not constitute a pre-requisite.

A related PhD topic can be foreseen.

References

[1] J. B. Lasserre and E. Pauwels. The empirical Christoffel function in Statistics and Machine Learning. 2017. `https://arxiv.org/abs/1701.02886`.