Exact Semidefinite Approximations of Reachable Sets

Research Context. Certification and validation of computational results is a major issue for modern sciences raising challenging problems at the interplay of mathematics and computational aspects of computer science. One can emphasize in this context several applications arising in the design of modern cyber-physical systems with a crucial need of *exact* certification. In particular, one tries to avoid incidents such as the Patriot missile crash in 1991, the FDIV Pentium bug in 1994 or more recently the collision of Google's self-driving car in 2016.

These issues give rise to many mathematical problems. Polynomial optimization (which consists in computing the infimum of a polynomial function under algebraic constraints) is one of the most important, difficult and challenging one. The emergence of this exciting new field goes back to the last decade and has led to striking developments from a cross fertilization between (real) algebraic geometry, applied mathematics, theoretical computer science and engineering.

Consider for instance the problem of minimizing $4x^4 + 4x^3y - 7x^2y^2 - 2xy^3 + 10y^4$ over $\mathbb{R}^2$. One way to certify that its minimum is 0 is to decompose this polynomial as a *sum of squares* (SOS), which is the core subject of study in real algebra. Here the decomposition is $(2xy + y^2)^2 + (2x^2 + xy - 3y^2)^2 \geq 0$. In general, one can compute such SOS decompositions by solving a *semidefinite program* (SDP) [1], which is a standard tool in applied mathematics and convex optimization. In SDP, one optimizes a linear function under the constraint that a given matrix is semidefinite positive, i.e. has only nonnegative eigenvalues. One particular issue arising while relying on SDP solvers is that they are often implemented using floating-point arithmetic, thus output only *approximate* certificates.

Given a dynamical polynomial system described by a continuous-time (differential) equation, the reachable set (RS) is the set of all states that can be reached from a set of initial conditions under general state constraints. This set appears in different fields such as optimal control, hybrid systems or program analysis. In general, computing or even approximating the RS is a challenge.

The ambitious goal of this project is to compute *exact* enclosures of RS for continuous-time systems.

Goals Preliminary work will consist of studying the exisiting algorithms to obtain exact SOS decompositions of nonnegative polynomials. In particular, the case of multivariate polynomial optimization has been recently handled in [2], thanks to a perturbation/compensation algorithm.

A promising research track is to design a similar algorithm to compute certified approximations for the reachable set of continuous-time polynomial systems. The first step shall be to derive a characterization of the RS, following the scheme developed for discrete-time systems [4].

Practical experiments shall be performed through implementing a tool within the Maple library `RealCertify` [3]. A further expected goal is to compare the performance with existing frameworks.

Working Context The internship will be co-advised by Victor Magron (CNRS) and Khalil Ghorbal (Hycomes group, INRIA Rennes, France). The Master student will be hosted either at Inria (Rennes, France) in the Hycomes group, or at LAAS CNRS (Toulouse, France) in the MAC team.

Required Skills Motivated candidates should hold a Bachelor degree and have a solid background in **either** optimization, computer arithmetics, real algebraic geometry or computer algebra. Good programming skills are also required. The candidates are kindly asked to send an e-mail with "M2 candidate" in the title, a CV and motivation letter to `vmagron@laas.fr` and `khalil.ghorbal@inria.fr`. Knowledge of French does not constitute a pre-requisite.

A related PhD topic can be foreseen.

References

[1] J.-B. Lasserre. Global Optimization with Polynomials and the Problem of Moments. *SIAM Journal on Optimization*, 11(3):796–817, 2001.

[2] V. Magron and M. S. E. Din. On Exact Polya and Putinar's Representations. In *Proceedings of ISSAC'18*. ACM, New York, NY, USA, 2018.

[3] V. Magron and M. S. E. Din. RealCertify: a Maple package for certifying non-negativity. In *Proceedings of ISSAC'18, **Best Software Demo Award***. ACM, New York, NY, USA, 2018.

[4] V. Magron, P.-L. Garoche, D. Henrion, and X. Thirioux. Semidefinite Approximations of Reachable Sets for Discrete-time Polynomial Systems, 2017. Submitted.