Modeling Physics with Differential-Algebraic Equations

Lecture 5

Multi-Mode DAEs

COMASIC (M2)
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source: Smart Grids, Les Echos 2015

```
model SimpleDrive
   ..Rotational.Inertia   Inertia1  (J=0.002);
   ..Rotational.IdealGear IdealGear1(ratio=100)
   ..Basic.Resistor       Resistor1 (R=0.2)
      ...
equation
   connect(Inertia1.flange_b, IdealGear1.flange_a);
   connect(Resistor1.n, Inductor1.p);
      ...
end SimpleDrive;
```

```
model Resistor
   package SIunits = Modelica.SIunits;
   parameter SIunits.Resistance R = 1;
   SIunits.Voltage v;
   ..Interfaces.PositivePin p;
   ..Interfaces.NegativePin n;
equation
   0 = p.i + n.i;
   v = p.v - n.v;
   v = R*p.i;
end Resistor;
```

```
type Voltage =
     Real(quantity="Voltage",
          unit    ="V");
```

```
connector PositivePin
   package SIunits = Modelica.SIunits;
   SIunits.Voltage      v;
   flow SIunits.Current i;
end PositivePin;
```

source:

[Peter Fritzson]

The Modelica Language and its Semantics

- Modelica Reference v3.3:

  "The semantics of the Modelica language is specified by means of a set of rules for translating any class described in the Modelica language to a flat Modelica structure"

- **the good:**
  - Semantics of continuous-time 1-mode Modelica models: Cauchy problem on the DAE resulting from the inlining of all components
  - Modelica supports *multi-mode* systems

```
1 = if g then x*x + y*y else y;
der(x) + x + y = 0;
when x <= 0 do reinit(x,1); end;
when y <= 0 do reinit(y,x); end;
```

- **the bad:** What about the semantics of multi-mode systems?
- **and ...:** Questionable simulations [Tim Bourke and Marc Pouzet]

```
model Scheduling
  Real x(start = 0);
  Real y(start = 0);
equation

  der(x) = 1;
  der(y) = x;

  when x >= 2 then
    reinit(x, -3 * y);
  end when;

  when x >= 2 then
    reinit(y, -4 * x);
  end when;

end Scheduling;
```

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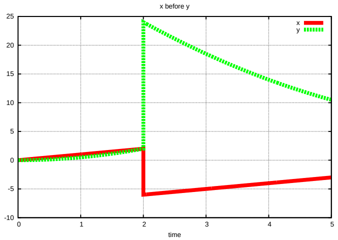

Causality: Modelica example

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  Real x(start = 0);
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  der(x) = 1;
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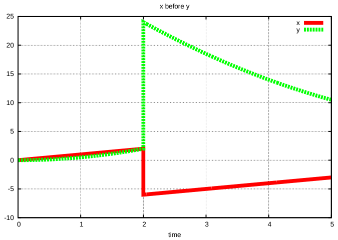
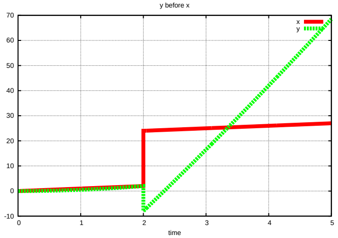
```





if Guard do Differential Equation

- **Guard**: predicate in the state variables and their **time derivatives**.
- **Differential Equation**: equation, **implicit** or explicit, in the state variables and their time derivatives.

`if` **Guard** `do` **Differential Equation**

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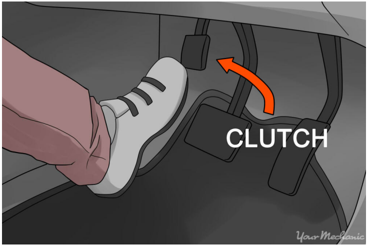
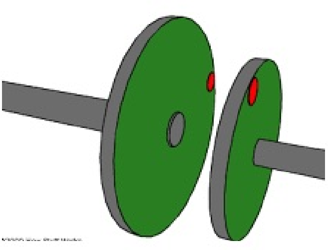
Goal

Structural analysis as a preprocessing step of **numerical simulation**.

Ideal Clutch





$$\begin{array}{rll}
\texttt{if } \top & \texttt{do} & J_1\dot{\omega}_1 = \tau_1 \qquad (e_1)\\
\texttt{if } \top & \texttt{do} & J_2\dot{\omega}_2 = \tau_2 \qquad (e_2)\\
\texttt{if } \gamma & \texttt{do} & \omega_1 - \omega_2 = 0 \qquad (e_3)\\
\texttt{if } \gamma & \texttt{do} & \tau_1 + \tau_2 = 0 \qquad (e_4)\\
\texttt{if } \neg\gamma & \texttt{do} & \tau_1 = 0 \qquad (e_5)\\
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\end{array}$$

- **State Variables**: the angular velocities ω_1 and ω_2
- γ is an input signal modelling the pedal's position

$$\begin{aligned}
&\texttt{if } \mathrm{T} \ \texttt{ do } \ J_1\dot{\omega}_1 = \tau_1 && (e_1)\\
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\end{array}
$$

Solve $Ax = b$. However A partially known at compile-time (may depend on parameters or state variables). Consider A with elements either zero or independent random variables varying in some neighborhood.

$$A = \begin{bmatrix} 1+\epsilon_1 & 0 & -1+\epsilon_2 & 0 \\ 0 & 1+\epsilon_3 & 0 & -1+\epsilon_4 \\ 1+\epsilon_5 & 1+\epsilon_6 & 0 & 0 \\ 0 & 0 & 1+\epsilon_7 & 1+\epsilon_8 \end{bmatrix}$$

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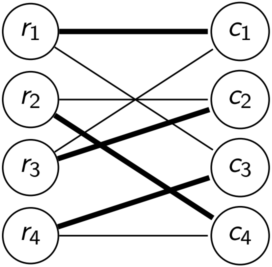

Thm A almost certainly nonsingular iff its incidence graph has a perfect matching

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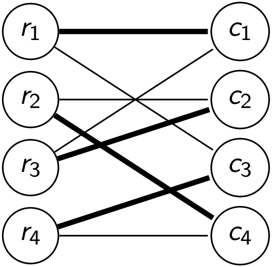

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Can be decided in time $O(n\sqrt{n})$.

System of Equations

$$
\begin{aligned}
f_1 :&\quad J_1\dot{\omega}_1 - \tau_1 &= 0\\
f_2 :&\quad J_2\dot{\omega}_2 - \tau_2 &= 0\\
f_3 :&\quad \omega_1 - \omega_2 &= 0\\
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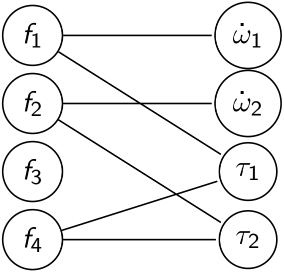
Bipartite Graph



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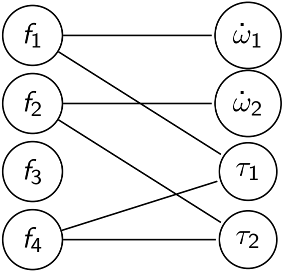

No perfect matching

Suppose a **consistent** initialization: $\omega_1 - \omega_2 = 0$

- f_3 is obviously satisfied
- A smooth solution that satisfies f_3 has to also satisfy $\dot{f}_3$
- $\dot{f}_3 : \dot{\omega}_1 - \dot{\omega}_2 = 0$

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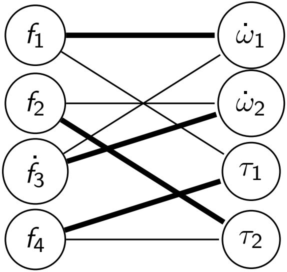

Matching Successful!

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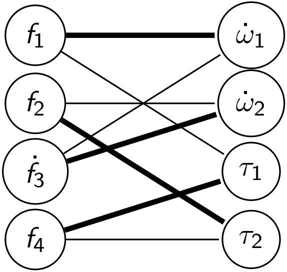
Incidence Matrix

	$\dot{\omega}_1$	$\dot{\omega}_2$	τ_1	τ_2
f_1	1	0	1	0
f_2	0	1	0	1
$\dot{f}_3$	1	1	0	0
f_4	0	0	1	1

Bipartite Graph



Matching Successful!

Implicit Function Theorem

Let $F(X, Y)$: n equations, where $|Y| = n$, $|X| = m$.
If $(u, v) \in \mathbb{R}^{m+n}$ is such that $F(u, v) = 0$ and $J = \frac{\partial F}{\partial Y}$ is nonsingular at (u, v), then there exists, in an open neighborhood U pf u, a unique set of functions G such that $v = G(u)$ and $F(w, G(w)) = 0$ for all $w \in U$

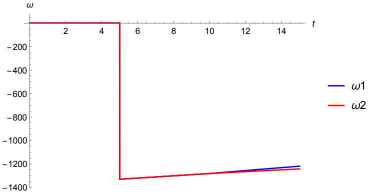
Structural NonSingularity

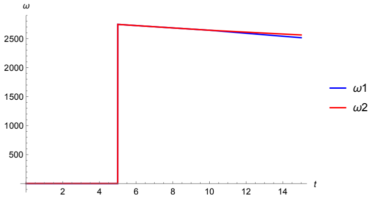
A square matrix is said to be structurally nonsingular if it remains almost everywhere nonsingular when its nonzero coefficients vary over some neighborhood.

Relation to the BTF Decomposition (Pantelides 1988)

The Jacobian $J = \frac{\partial F}{\partial Y}$ is structurally nonsingular if and only if G_F (the incidence graph related to F) can be decomposed in a BTF form.

- Dymola **crashes** with a division by zero
- Mathematica treats resets as initializations (nondeterministic behavior)





The solution may be discontinuous when $\gamma : \mathrm{F} \to \mathrm{T}$ because of the additional constraint $\omega_1 - \omega_2 = 0$

Problem 1 How to handle overdetermined systems ?

- The angular velocities ω_1 and ω_2 are **known**
- γ switches to T (the driver engages the clutch)

$\omega_1 - \omega_2 = 0$ is **enforced**

- The system **becomes** overdetermined
- The solution is not smooth and even discontinuous

Problem 2 What is the meaning of the derivatives ?

Some equations must hold for $\gamma = \mathrm{T}$ and $\gamma = \mathrm{F}$.

$$\begin{array}{llll} \texttt{if } \mathrm{T} & \texttt{do} & J_1\dot{\omega}_1 = \tau_1 & (e_1) \\ \texttt{if } \mathrm{T} & \texttt{do} & J_2\dot{\omega}_2 = \tau_2 & (e_2) \end{array}$$

- What is the **meaning** of derivatives when $\gamma : \mathrm{F} \to \mathrm{T}$?
- How to compute the **reset values** ?

Causality Principle

The additional constraints are

- **caused** by (consequence of) the **current** status, and
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t : **present**

$\omega_1(t) - \omega_2(t) \neq 0$

$\omega_1(t+\delta) - \omega_2(t+\delta) = 0$

$t+\delta\ ,\ 0 < \delta << 1$: **immediate future**

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$t + \delta\ ,\ 0 < \delta << 1$: **immediate future**

$\delta \in {}^\star\mathbb{R}$ is a **positive infinitesimal**

- $\delta = \langle \delta_1, \delta_2, \ldots \rangle$
- $\delta_i \in \mathbb{R}$
- Not necessarily convergent
- $\langle 1, \frac{1}{2}, \frac{1}{3}, \ldots \rangle$ is a (positive) infinitesimal
- $r = \langle r, r, r, \ldots \rangle$, $r \in \mathbb{R}$
- Functions over the reals can be *internalized*
- $x(\langle t_1, t_2, \ldots \rangle) = \langle x(t_1), x(t_2), \ldots \rangle$

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Let $\delta \in {}^\star\mathbb{R}$ be a non zero infinitesimal.

$$\frac{x(t+\delta) - x(t)}{\delta}$$

Proposition

A real function x is differentiable at t if and only if there exists a real number b such that

$$\frac{x(t+\epsilon) - x(t)}{\epsilon} \sim b$$

for any non zero infinitesimal ϵ.

$\dot{x}$ is replaced by $\dfrac{x(t+\delta) - x(t)}{\delta} = \dfrac{x^\bullet - x}{\delta}$

- **Shift forward** (when needed)
- **Formal substitution** of time derivatives into difference quotient.

$$\begin{array}{rlll} \texttt{if}\ \mathrm{T} & \texttt{do} & J_1\dot{\omega}_1 = \tau_1 & (e_1) \\ \texttt{if}\ \mathrm{T} & \texttt{do} & J_2\dot{\omega}_2 = \tau_2 & (e_2) \\ \texttt{if}\ \gamma & \texttt{do} & \omega_1 - \omega_2 = 0 & (e_3) \\ \texttt{if}\ \gamma & \texttt{do} & \tau_1 + \tau_2 = 0 & (e_4) \\ \texttt{if}\ \neg\gamma & \texttt{do} & \tau_1 = 0 & (e_5) \\ \texttt{if}\ \neg\gamma & \texttt{do} & \tau_2 = 0 & (e_6) \end{array}$$

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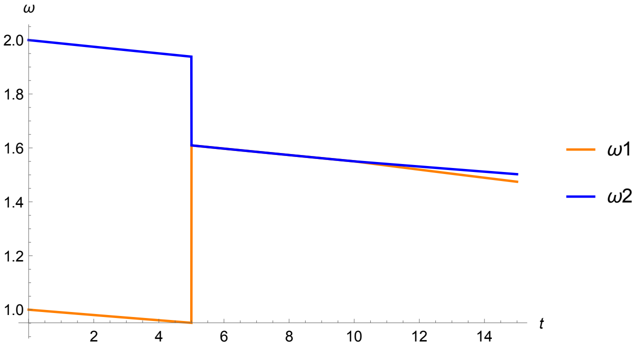
$$\omega_1^\bullet = \omega_2^\bullet = \frac{J_1\omega_1 + J_2\omega_2}{J_1 + J_2}$$

Standardization

- Automated procedure for a class of systems
- Generalization remains a challenge

Expected Simulation



- Based on **abstract semantics**
- **No** actual **computation** takes place

Abstract Domain

$$D = \{\mathrm{I}, \mathrm{U}, \mathrm{F}, \mathrm{T}\} \quad \text{with} \quad \mathrm{I} < \mathrm{U} < \mathrm{F}, \mathrm{T}$$

"I", **irrelevant**, denotes dead code

A **guard** may be

- not evaluated (U)
- evaluated ($\{\mathrm{T},\mathrm{F}\}$)

A **guard** may be

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A **variable** may be

- undefined (U)
- or defined (T)

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A **variable** may be

- undefined (U)
- or defined (T)

A **guarded equation** may be

- disabled (F)
- enabled and its equation is not yet used (U)
- enabled and its equation is used/known (T)

- A **status** is a map $\sigma : V \to D$
- V contains **variables**, **guards** and **guarded equations** altogether (up to a statically determined finite order)

Coherence

- An equation can not be used before its guard evaluates to true
- A guard can not be evaluated before evaluating all its incident variables

Successful Status

- All non irrelevant guarded equations have either the value T or F
- No leading variable has the value U

Micro-step (Algorithm 5)

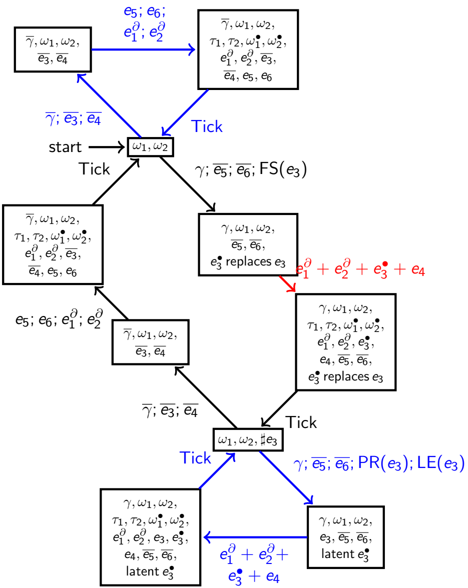
- Takes a coherent status and returns a coherent status
- Updates the values of a subset of V from U to T or F and from I to U

Run

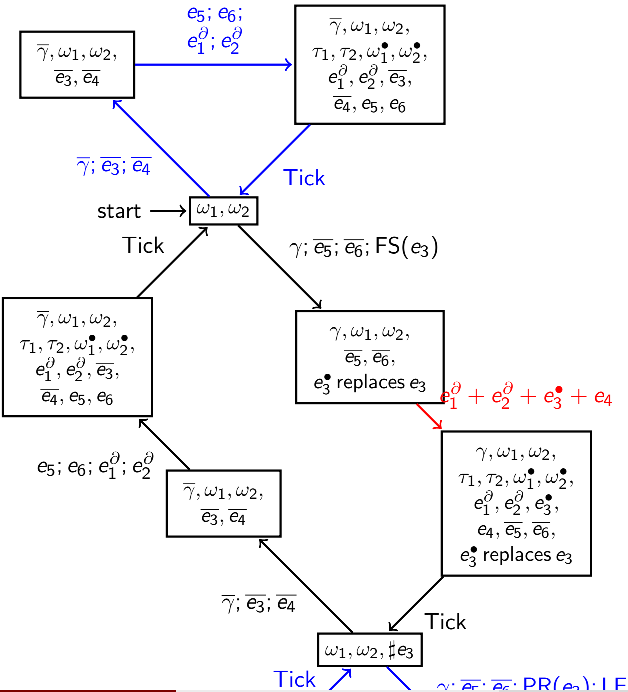
A finite sequence of statuses

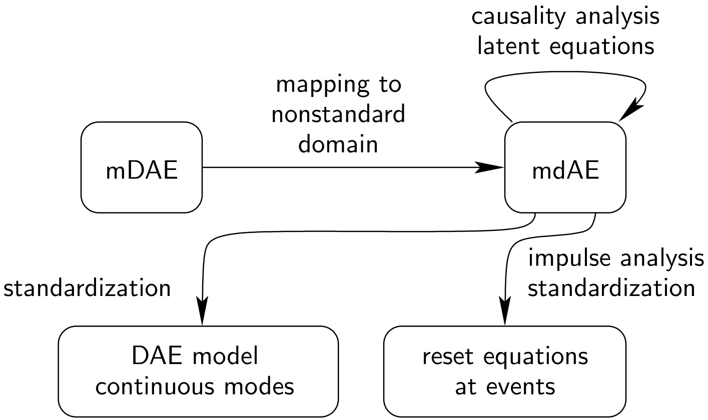
$$\sigma_0 < \sigma_1 < \cdots < \sigma_K$$

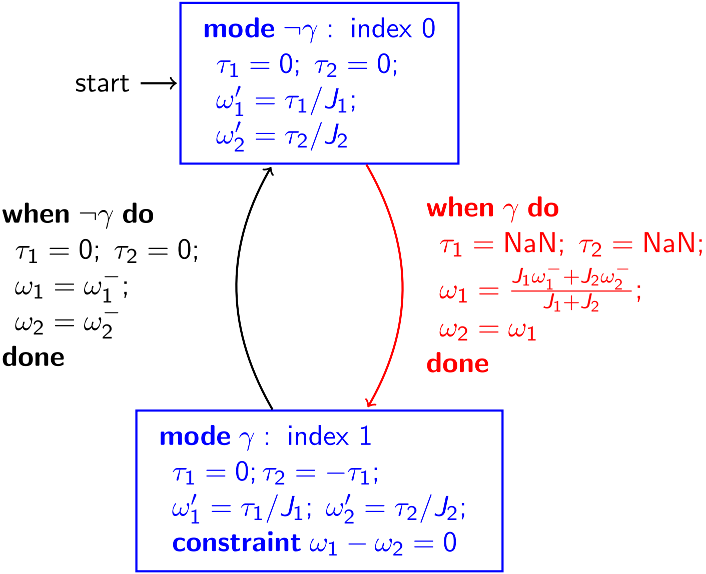
such that (σ_i, σ_{i+1}) is a micro-step.
A run is **successful** if its last status (σ_K) is successful.



$\overline{\gamma}, \omega_1, \omega_2, \overline{e_3}, \overline{e_4}$
$e_5; e_6; e_1^\partial; e_2^\partial$
$\overline{\gamma}, \omega_1, \omega_2, \tau_1, \tau_2, \omega_1^\bullet, \omega_2^\bullet, e_1^\partial, e_2^\partial, \overline{e_3}, \overline{e_4}, e_5, e_6$
$\overline{\gamma}; \overline{e_3}; \overline{e_4}$
Tick
start
ω_1, ω_2
Tick
$\gamma; \overline{e_5}; \overline{e_6}$; FS($e_3$)
$\overline{\gamma}, \omega_1, \omega_2, \tau_1, \tau_2, \omega_1^\bullet, \omega_2^\bullet, e_1^\partial, e_2^\partial, \overline{e_3}, \overline{e_4}, e_5, e_6$
$\gamma, \omega_1, \omega_2, \overline{e_5}, \overline{e_6}$, $e_3^\bullet$ replaces e_3
$e_1^\partial + e_2^\partial + e_3^\bullet + e_4$
$e_5; e_6; e_1^\partial; e_2^\partial$
$\overline{\gamma}, \omega_1, \omega_2, \overline{e_3}, \overline{e_4}$
$\gamma, \omega_1, \omega_2, \tau_1, \tau_2, \omega_1^\bullet, \omega_2^\bullet, e_1^\partial, e_2^\partial, e_3^\bullet, e_4, \overline{e_5}, \overline{e_6}$, $e_3^\bullet$ replaces e_3
$\overline{\gamma}; \overline{e_3}; \overline{e_4}$
Tick
$\omega_1, \omega_2, \sharp e_3$
Tick
$\gamma; \overline{e_5}; \overline{e_6}$; PR($e_3$); LE($e_3$)
$\gamma, \omega_1, \omega_2, \tau_1, \tau_2, \omega_1^\bullet, \omega_2^\bullet, e_1^\partial, e_2^\partial, e_3, e_3^\bullet, e_4, \overline{e_5}, \overline{e_6}$, latent $e_3^\bullet$
$e_1^\partial + e_2^\partial + e_3^\bullet + e_4$
$\gamma, \omega_1, \omega_2, e_3, \overline{e_5}, \overline{e_6}$, latent $e_3^\bullet$


$\overline{\gamma}, \omega_1, \omega_2, \overline{e_3}, \overline{e_4}$
$e_5; e_6; e_1^\partial; e_2^\partial$
$\overline{\gamma}, \omega_1, \omega_2, \tau_1, \tau_2, \omega_1^\bullet, \omega_2^\bullet, e_1^\partial, e_2^\partial, \overline{e_3}, \overline{e_4}, e_5, e_6$
$\overline{\gamma}; \overline{e_3}; \overline{e_4}$
Tick
start
ω_1, ω_2
Tick
$\gamma; \overline{e_5}; \overline{e_6}; \mathrm{FS}(e_3)$
$\overline{\gamma}, \omega_1, \omega_2, \tau_1, \tau_2, \omega_1^\bullet, \omega_2^\bullet, e_1^\partial, e_2^\partial, \overline{e_3}, \overline{e_4}, e_5, e_6$
$\gamma, \omega_1, \omega_2, \overline{e_5}, \overline{e_6}$, $e_3^\bullet$ replaces e_3
$e_1^\partial + e_2^\partial + e_3^\bullet + e_4$
$e_5; e_6; e_1^\partial; e_2^\partial$
$\overline{\gamma}, \omega_1, \omega_2, \overline{e_3}, \overline{e_4}$
$\gamma, \omega_1, \omega_2, \tau_1, \tau_2, \omega_1^\bullet, \omega_2^\bullet, e_1^\partial, e_2^\partial, e_3^\bullet, e_4, \overline{e_5}, \overline{e_6}$, $e_3^\bullet$ replaces e_3
$\overline{\gamma}; \overline{e_3}; \overline{e_4}$
Tick
$\omega_1, \omega_2, \sharp e_3$
Tick


causality analysis
latent equations
mapping to
nonstandard
domain
mDAE
mdAE
standardization
impulse analysis
standardization
DAE model
continuous modes
reset equations
at events



mode $\neg\gamma$: index 0
$\tau_1 = 0;\ \tau_2 = 0;$
$\omega_1' = \tau_1/J_1;$
$\omega_2' = \tau_2/J_2$
start $\longrightarrow$
when $\neg\gamma$ **do**
$\tau_1 = 0;\ \tau_2 = 0;$
$\omega_1 = \omega_1^-;$
$\omega_2 = \omega_2^-$
done
when γ **do**
$\tau_1 = \text{NaN};\ \tau_2 = \text{NaN};$
$\omega_1 = \frac{J_1\omega_1^- + J_2\omega_2^-}{J_1+J_2};$
$\omega_2 = \omega_1$
done
mode γ : index 1
$\tau_1 = 0; \tau_2 = -\tau_1;$
$\omega_1' = \tau_1/J_1;\ \omega_2' = \tau_2/J_2;$
constraint $\omega_1 - \omega_2 = 0$

- A. Benveniste, B. Caillaud, H. Elmqvist, K. Ghorbal, M. Otter, and M. Pouzet. Structural analysis of multi-mode DAE systems. In HSCC, pages 253–263. ACM, 2017.
- A. Benveniste, B. Caillaud, M. Pouzet, H. Elmqvist, and M. Otter. Structural Analysis of Multi-Mode DAE Systems. Research Report RR-8933, Inria, July 2016.