Master Sciences Informatiques
Advanced Semantics (ASM)
Final Exam, January 2020

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1 Operational Semantics

Recall that the *call by value* strategy evaluates the arguments of a function before passing them to the function. Using such strategy

1. Give a big-step semantics for the λ-calculus
2. Give a small-step semantics for the λ-calculus
3. Give a reduction strategy for the λ-calculus
4. Give an abstract machine semantics for the λ-calculus
5. Select any two semantics from 1. to 4. above and show their equivalence

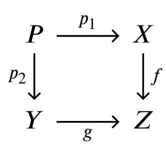
2 Categorical Semantics

A. Monadic Semantics

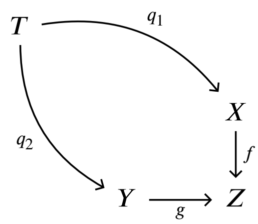
1. Suppose you have a '`let`' binder in your language, that is '`let x = t in u`' is a term (`x` is a variable and `t,u` are terms). Is it possible to give a general big-step operational semantics for such binder without discussing the possible reduction of the term `u` once the variable `x` is substituted by `t`? Explain.
2. Recall what a Kleisli triple is and describe concisely how does it capture the semantics of a particular computation.
3. Explain why and how types are used in a Kleisli triple and give the semantics of the `let` binder with respect to a given computation.

B. Category Theory: Pullbacks

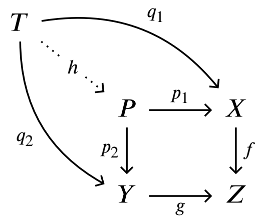
A commutative diagram of a category $\mathscr{C}$



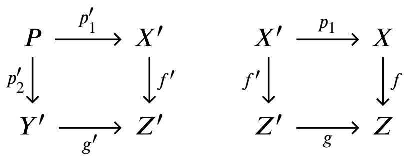
is called a *pullback* diagram (*produit fibré* dans la langue de Molière) when the following property holds: for every commutative diagram



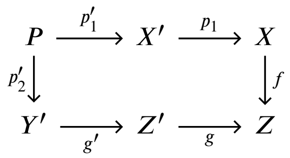
there exists a unique morphism $h : T \to P$ such that the following diagram commutes.



1. Given $f : X \to Z$ and $g : Y \to Z$, give explicitly a pullback in the category of sets (objects are sets and morphisms are functions).
2. Given two pullback diagrams



show that we can construct another pullback by gluing together the given pullbacks:



3. A morphism $m : A \to B$ in $\mathscr{C}$ is a *monomorphism* (or mono) if we can remove it from the left for every pair of morphisms $f, g : X \to A$, that is

$$m \circ f = m \circ g \implies f = g \ .$$

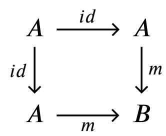
Show that monomorphisms correspond to injective functions in the category of sets.

4. Similarly, a morphism $e : A \to B$ in $\mathscr{C}$ is an *epimorphism* (or epi) if we can remove it from the right for pair of morphisms $f, g : B \to Y$, that is

$$f \circ e = g \circ e \implies f = g \ .$$

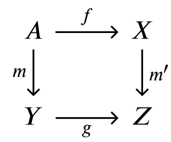
Show that epimorphisms correspond to surjective functions in the category of sets.

5. Show that a morphism m is mono if and only if the following commutative diagram is a pullback.



What is the meaning of this property in the category of sets?

6. Show that every pullback diagram



satisfies $m : A \to Y$ and $m' : X \to Z$ are mono. Does the converse hold? Can you think of a counter-example in the category of sets?